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Fuzzy economic order time models with random demand

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A B S T R A C T

Inventory models in which the time period of sales is a decision variable are considered in fuzzy environments. Arrival of customers and the number of customers in the planning period are both random. Several cases of the model with one or more of the components fuzzy are discussed. Models with trapezoidal and triangular fuzzy numbers are considered. Optimum policy of the models are derived and numerical examples are provided in order to ascertain the sensitiveness in the decision variable with respect to fuzziness in the components.

1. Introduction

The first scientific approach to inventory management problem was the Harris–Wilson method popularly known as the economic order quantity (EOQ) formula. In the EOQ inventory system, the input is made in equal sizes against continuous withdrawal of items at a constant rate. The costs to be considered are the setup cost, production cost and inventory holding cost. The EOQ formula gives the order quantity so as to meet customer service levels while minimizing the total inventory cost. This formula is generally recommended in problems where demand is relatively steady. Being very simple to implement, stock manufacturers use the EOQ formula for fixing the quantity to be produced while the stock distributors use it for fixing the quantity to be purchased. A number of authors have considered several variations in the standard EOQ model. An EOQ model for items with an exponentially decaying inventory was investigated by Ghare and Schrader [6]. Covert and Philip [5] and Tadikamalla [17] studied the EOQ model for items with weibull and gamma type deterioration respectively. Liberatore [11] developed an EOQ model where the uncertainties in the lead time is represented stochastically. Goyal [7] established a single item EOQ model under the condition of permissible delay in payments. Hariga [9] described an EOQ model for deteriorating items with shortage and time varying demand. An EOQ inventory model for perishable items was developed by Padmanabhan and Vrat [13] under stock dependent selling rate. Recently, Chen [4] studied an EOQ model under random demand.

One or more components of an inventory model often appear to be vague and imprecise and hence for getting realistic models all such components are to be represented by fuzzy sets. A number of researchers have applied the fuzzy set concepts to deal with the EOQ problems. Park [14] developed a fuzzy EOQ model where the ordering and holding costs are represented by trapezoidal fuzzy numbers. Vujoevia et al. [19] considered EOQ model under fuzzy cost components. Roy and Maiti [15] rewrote the problem of classic EOQ into a form of nonlinear programming problem and introduced fuzziness both in the objective function and constraints of storage area. Lee and Yao [10] developed an EOQ model where the demand and order

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quantities are represented by fuzzy sets. Mandal and Maiti [12] described multi-item EOQ models with objective function, constraints and inventory costs are represented by fuzzy sets. An EOQ model without backorders with fuzzy total demand and fuzzy storage cost was discussed by Yao and Chiang [22]. Chang [1] studied an EOQ model with imperfect quality items where fuzziness is introduced in the defective rate and annual demand. Yadavalli et al. [21] considered a multi-item EOQ model with fuzzy cost. Wanga et al. [20] developed an EOQ problem with imperfect quality item characterized as a fuzzy random variable while the setup and holding costs as fuzzy variables. Shiang [16] studied an EOQ model under fuzzy demand quantity and fuzzy cost. An EOQ model for perishable items with fuzzy partial backlogging factor and fuzzy deterioration rate was developed by Halim et al. [8].

In this paper, an inventory system in which the time period of sales is the decision variable proposed by Chen [4] is reconsidered assuming the components of the model as fuzzy sets. Arrival of customers and the number of customers in the planning period are both random. The author derived the optimal length of the selling period so as to minimize the average inventory cost per unit time. It was pointed out that the number of customers arriving in the planning time period in Chen's model is equivalent to the order quantity in the traditional EOQ model. As such, Chen's model can be considered as an EOQ model with quantity representing the time period, hence we name it as the economic order time (EOT) model. Section 2 briefly presents the EOT model under random demand and random purchasing time. The Lagrangian method of optimization is described in Section 3. Section 4 reviews the basic concepts of fuzzy set theory. The fuzzy equivalent of the EOT model with all components fuzzy is described in Section 5. Section 6 describes the fuzzy model with crisp time period and model under fuzzy mean arrival rate is given in Section 7. The defuzzified value of fuzzy cost function is derived by adopting the graded mean integration representation of fuzzy numbers. The last Section presents the numerical illustrations of the developed models followed by some concluding remarks.

2. EOT model with random demand

The basic EOQ model determines the economic order quantity which minimize the total cost based on the assumptions that the total demand is constant and shortage is not permitted. The Wilson–Harry's optimal inventory size formula is given by

$$Q = \sqrt{\frac{2Ka}{h}}, \tag{1}$$

where K, h and a are the setup cost, holding cost per unit and total demand per unit time, respectively. However, the traditional EOQ model seems to be effective if the demand and cost components are completely known. Since the total demand is usually uncertain, it is more realistic to replace the constant demand by the expected value of the total demand. Chen [4] reconstructed the EOQ model based on the random demand.

The following notations and assumptions are used.

Notations

- $[0, t]$ the selling period, t being the length of the given period.
- C the purchasing cost per unit of goods.
- K the setup cost.
- h the unit holding cost per unit time.

Assumption.

(i) The purchasing time of the customer is a uniform $(0, t)$ random variable.
(ii) The number of customers that arrive in the unit time interval (quantity of demand) follows a Poisson distribution with mean arrival rate per unit time θ.
(iii) The total number of customers in the interval $[0, t]$ follows a Poisson distribution with parameter θt.
(iv) The purchasing times of the customers are independently and identically distributed and they are mutually independent with the total number of customers in the selling period.

The expected total cost in the whole period $[0, t]$ is given by Chen [4]

$$Z(t) = K + C\theta\ t + \frac{h\theta t^2}{2}. \tag{2}$$

The expected total cost per unit time is

$$\mathscr{A}(t) = \frac{Z(t)}{t}, \quad \frac{K}{t} + C\theta + \frac{h\theta t}{2}. \tag{3}$$

The objective is to find the length of the selling period t which minimize the average cost per unit time. The necessary condition for Eq. (3) to be minimum, $\frac{\partial \mathscr{A}(t)}{\partial t} = 0$ implies that

$$t = \sqrt{\frac{2K}{\theta h}}, \tag{4}$$

at which

$$\frac{\partial^2 \mathscr{A}(t)}{\partial t^2} = \frac{2K}{t^3} > 0. \tag{5}$$

Hence, t given by Eq. (4) minimizes the average cost in Eq. (3). It is noted that variations in the purchasing cost will not affect the optimum time period.

The above model can be considered as an EOQ model. Let N_t^* denote the number of customers arriving in the time interval $(0, t)$ when t equals the optimum value given in Eq. (4). Since the probability distribution of N_t^* is assumed to be Poisson, expected value of N_t^* or expected quantity of demand is given by

$$\mathscr{E}(N_t^*) = \theta t = \sqrt{\frac{2K\theta}{h}}. \tag{6}$$

It should be noted that, Eq. (6) is also equivalent to the result of traditional EOQ formula, provided that the quantity of total demand a per unit time in Eq. (1) is replaced by θ, the expected demand per unit time. Thus $\mathscr{E}(N_t^*)$ is the expected economic order quantity.

The fuzzy equivalent of the above model is described in Section 5. In the following section, the Lagrangian optimization technique, needed to solve the fuzzy model is described.

3. Lagrangian optimization method

The techniques for identifying the stationary points of a nonlinear programming problem subject to inequality constraints is based on the Lagrangian method. The Karush–Kuhn–Tucker conditions are necessary and sufficient conditions for minimization problem if both the objective function and solution space are convex. In the case of minimization problem with non negative constraints, the solution space is convex if the constraint function is concave and the Lagrangian multipliers are non negative. In such case, the Lagrangian function must be convex and the resulting stationary point yields a global constrained minimum. We adopt the extended Lagrangian method to solve the non linear programming problem with inequality constraints. This method is described in several standard text books, Taha [18] is one of the latest references. The general idea of extended Lagrangian procedure is that if the unconstrained optimum problem does not satisfy all the constraints, the constrained optimum must occur at the boundary point of the solution space. This means that at least one constraint must be satisfied in equation form. In this case, at the optimal point, the Karush–Kuhn–Tucker necessary conditions indicate that the negative of the gradient of the objective function (represent the direction of steepest descent) must be expressible as a positive linear combination (the coefficients are the Lagrangian multipliers) of the gradient of the active constraints.

The best to be hoped for using the extended Lagrangian method is a good feasible solution. If the problem possess a unique constrained optimum, the procedure can be rectified to locate the global optimum.

For the minimization problem, Minimize $Y = f(X)$ subject to $g_i(X) \geqslant 0, i = 1, 2, \ldots, M$, where the nonnegativity constraints $X \geqslant 0$, if any, are also included in the M constraints, the procedure of extension of Lagrangian method involves the following steps.

Step (i): Solve the unconstrained problem Minimize $Y = f(X)$. If the resulting optimum satisfies all the constraints, stop the procedure. Otherwise set the number of constraints $\mathscr{K} = 1$ and go to step (ii).
Step (ii): Activate any $\mathscr{K}$ constraints by converting them into equalities and minimize $f(X)$ subject to the $\mathscr{K}$ active constraints by the Lagrangian method. If the resulting solution is feasible with respect to the remaining constraints, stop; it is a local optimum. Otherwise, take another set of $\mathscr{K}$ constraints and repeat the step. If all sets of active constraints taken at a time are considered without encountering a feasible solution, go to step (iii).
Step (iii): If $\mathscr{K} = M$, stop; no feasible solution exists. Otherwise set $\mathscr{K} = \mathscr{K} + 1$ and go to step (ii).

Some preliminary concepts of fuzzy set theory required in the development of our models are described below.

4. Fuzzy set

In a universe of discourse X, a fuzzy subset $\widetilde{A}$ on X is defined by the membership function $\mu_{\widetilde{A}}(x)$ which maps each element x in X to a real number in the interval $[0, 1]$. $\mu_{\widetilde{A}}(x)$ denotes the grade or degree of membership and it is usually denoted as $\mu_{\widetilde{A}} : X \to [0, 1]$. A fuzzy set is said to be normal if the largest grade obtained by any element in that set is 1. That is, there must exist at least one x for which $\mu_{\widetilde{A}}(x) = 1$. The support of $\widetilde{A}$ is defined as the crisp set that contains all elements of X that have non zero membership grades. A fuzzy set $\widetilde{A}$ on X is convex iff $\mu_{\widetilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geqslant \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2))$ for all $x_1, x_2 \varepsilon X$ and for $\lambda \varepsilon [0, 1]$, where min denotes the minimum operator.

4.1. Fuzzy number

A fuzzy number is a fuzzy subset of the real line which is both normal and convex. In addition, the membership function of a fuzzy number must be piecewise continuous.

The membership function of a fuzzy number $\widetilde{A}$ is usually represented as

$$\begin{aligned} \mu_{\widetilde{A}}(x) &= l(x), \quad x < m, \\ &= 1, \quad m \leqslant x \leqslant n, \\ &= u(x), \quad x > n, \end{aligned} \tag{7}$$

where $l(x)$ is continuous from the right, strictly increasing for $x < m$ and there exist $m_1 < m$ such that $l(x) = 0$ for $x \leqslant m_1$ and $u(x)$ is continuous from the left, strictly decreasing for $x > n$ and there exist $n_1 \geqslant n$ such that $u(x) = 0$ for $x \geqslant n_1$. $l(x)$ and $u(x)$ are called the left and right reference functions, respectively.

The fuzzy number $\widetilde{A}$ is said to be a trapezoidal fuzzy number if it is fully determined by (a_1, a_2, a_3, a_4) of crisp numbers such that $a_1 < a_2 < a_3 < a_4$, with membership function, representing a trapezoid, of the form

$$\begin{aligned} \mu_{\widetilde{A}}(x) &= \frac{x - a_1}{a_2 - a_1}, \quad a_1 \leqslant x \leqslant a_2, \\ &= 1, \quad a_2 \leqslant x \leqslant a_3, \\ &= \frac{x - a_4}{a_3 - a_4}, \quad a_3 \leqslant x \leqslant a_4, \\ &= 0, \quad \text{otherwise}, \end{aligned} \tag{8}$$

where a_1, a_2, a_3 and a_4 are the lower limit, lower mode, upper mode and upper limit respectively of the fuzzy number $\widetilde{A}$.

When $a_2 = a_3$, the trapezoidal fuzzy number becomes a triangular fuzzy number.

4.2. Fuzzy arithmetic operations

Some fuzzy arithmetic operations under the functional principle Chen [2] for trapezoidal fuzzy numbers are given below. Let $\widetilde{A_1} = (a_{11}, a_{12}, a_{13}, a_{14})$ and $\widetilde{A_2} = (a_{21}, a_{22}, a_{23}, a_{24})$ be two trapezoidal fuzzy numbers. Then

(i) Addition

$$\widetilde{A_1} + \widetilde{A_2} = (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}).$$

(ii) Multiplication

If $a_{11}, a_{21}, a_{12}, a_{22}, a_{13}, a_{23}, a_{14}$ and a_{24} are all positive real numbers, then

$$\widetilde{A_1} * \widetilde{A_2} = (a_{11}a_{21}, a_{12}a_{22}, a_{13}a_{23}, a_{14}a_{24}).$$

(iii) Subtraction

$$-\widetilde{A_2} = (-a_{24}, -a_{23}, -a_{22}, -a_{21}), \quad \widetilde{A_1} - \widetilde{A_2} = (a_{11} - a_{24}, a_{12} - a_{23}, a_{13} - a_{22}, a_{14} - a_{21}).$$

(iv) Division

If $a_{11}, a_{21}, a_{12}, a_{22}, a_{13}, a_{23}, a_{14}$ and a_{24} are all positive real numbers, then

$$\frac{1}{\widetilde{A_2}} = \widetilde{A_2}^{-1} = \left(\frac{1}{a_{24}}, \frac{1}{a_{23}}, \frac{1}{a_{22}}, \frac{1}{a_{21}}\right),$$

$$\frac{\widetilde{A_1}}{\widetilde{A_2}} = \left(\frac{a_{11}}{a_{24}}, \frac{a_{12}}{a_{23}}, \frac{a_{13}}{a_{22}}, \frac{a_{14}}{a_{21}}\right).$$

(v) Scalar multiplication

Let k be a real number, then for $k \geqslant 0, k\widetilde{A_1} = (ka_{11}, ka_{12}, ka_{13}, ka_{14})$ and $k < 0, k\widetilde{A_1} = (ka_{14}, ka_{13}, ka_{12}, ka_{11})$.

4.3. Defuzzification

In order to draw ultimate conclusions for decision making, the fuzzy results are to be converted into crisp values. The method of extracting crisp results from the fuzzy models is known as defuzzification. In this paper, we adopt the graded mean integration representation introduced by Chen and Hseih [3] for defuzzification. The extension principle to find the membership function of fuzzy total cost function is though direct, is not simple in most of the cases. As the membership function does not change under fuzzy arithmetic operations, it is possible to evaluate the defuzzified value directly by graded mean integration method through arithmetic operations. It is more reasonable to discuss the grade of each point of support set of fuzzy number for representing the fuzzy number. Chen and Hseih's method is effective in the sense that it grades as the degree of each point of support set of fuzzy number and it is possible to measure the degree of similarity between fuzzy numbers in terms of graded mean integration values.

For the fuzzy number $\widetilde{A}$ in Eq. (7), let l^{-1} and u^{-1} denote the inverse functions of l and u, respectively. The graded mean γ level value of $\widetilde{A}$ is $\frac{1}{2}(\gamma(l^{-1}(\gamma)+u^{-1}(\gamma))$. The graded mean representation of $\widetilde{A}$ is given by

$$\varrho(\widetilde{A}) = \frac{1}{\int_0^1 \gamma d\gamma}\int_0^1 \left(\frac{l^{-1}(\gamma)+u^{-1}(\gamma)}{2}\right)\gamma d\gamma. \tag{9}$$

For the trapezoidal fuzzy number $\widetilde{A}=(a_1,a_2,a_3,a_4)$, $l^{-1}(\gamma)=a_1+(a_2-a_1)\gamma$ and $u^{-1}(\gamma)=a_4-(a_4-a_3)\gamma$.

The graded mean representation of trapezoidal fuzzy number $\widetilde{A}=(a_1,a_2,a_3,a_4)$ from Eq. (9) is given by

$$\varrho(\widetilde{A}) = \frac{a_1+2a_2+2a_3+a_4}{6}. \tag{10}$$

In the case of triangular fuzzy number $\widetilde{A}=(a_1,a_2,a_3)$,

$$\varrho(\widetilde{A}) = \frac{a_1+4a_2+a_3}{6}. \tag{11}$$

The Sections 5–7 describe different cases of the EOT model of Section 2 in fuzzy environments. The model with all the components fuzzy is considered in Section 5 while the one with crisp time period is developed in Section 6.

5. Fuzzy EOT model with fuzzy time period

In this section, we consider model in Section 2 with all the five parameters C, K, t, h and θ as fuzzy and they are represented by trapezoidal fuzzy numbers, as follows:

$$\widetilde{C}=(C-\kappa_1,C-\kappa_2,C+\kappa_3,C+\kappa_4),\quad \widetilde{K}=(K-\kappa_5,K-\kappa_6,K+\kappa_7,K+\kappa_8),$$
$$\tilde{h}=(h-\kappa_9,h-\kappa_{10},h+\kappa_{11},h+\kappa_{12}),\quad \tilde{t}=(t-v_1,t-v_2,t+v_3,t+v_4),\quad \text{and}$$
$$\tilde{\theta}=(\theta-v_5,\theta-v_6,\theta+v_7,\theta+v_8).$$

$\kappa_i, i=1,2,\ldots,12$ and $v_i, i=1,2,\ldots,8$ are arbitrary positive numbers which satisfy $\kappa_1>\kappa_2, \kappa_3<\kappa_4, \kappa_5>\kappa_6, \kappa_7<\kappa_8, \kappa_9>\kappa_{10}, \kappa_{11}<\kappa_{12}, v_1>v_2, v_3<v_4, v_5>v_6$ and $v_7<v_8$.

Fuzzy expected cost per unit time from Eq. (3) is given by

$$\widetilde{\mathscr{A}}(\tilde{t}) = \frac{\widetilde{K}}{\tilde{t}} + \tilde{\theta}\widetilde{C} + \frac{\tilde{h}\tilde{\theta}\tilde{t}}{2}, \tag{12}$$

where $\frac{\widetilde{K}}{\tilde{t}}, \tilde{\theta}\widetilde{C}$ and $\tilde{h}\tilde{\theta}\tilde{t}$ are given by the fuzzy arithmetic operations in Section 4 as, the trapezoidal fuzzy numbers

$$\frac{\widetilde{K}}{\tilde{t}} = \left(\frac{K-\kappa_5}{t+v_4},\frac{K-\kappa_6}{t+v_3},\frac{K+\kappa_7}{t-v_2},\frac{K+\kappa_8}{t-v_1}\right), \tag{13}$$

$$\tilde{\theta}\widetilde{C} = ((\theta-v_5)(C-\kappa_1),(\theta-v_6)(C-\kappa_2),(\theta+v_7)(C+\kappa_3),(\theta+v_8)(C+\kappa_4)), \tag{14}$$

and

$$\tilde{h}\tilde{\theta}\tilde{t} = ((\theta-v_5)(h-\kappa_9)(t-v_1),(\theta-v_6)(h-\kappa_{10})(t-v_2),(\theta+v_7)(h+\kappa_{11})(t+v_3),(\theta+v_8)(h+\kappa_{12})(t+v_4)). \tag{15}$$

Using the above Eqs. (13)–(15) in Eq. (12), we have the trapezoidal fuzzy number

$$\widetilde{\mathscr{A}}(\tilde{t}) = (\mathscr{A}_1,\mathscr{A}_2,\mathscr{A}_3,\mathscr{A}_4), \tag{16}$$

where

$$\mathscr{A}_1 = \tfrac{K-\kappa_5}{t+v_4}+(\theta-v_5)(C-\kappa_1)+\tfrac{(\theta-v_5)(h-\kappa_9)(t-v_1)}{2},$$
$$\mathscr{A}_2 = \tfrac{K-\kappa_6}{t+v_3}+(\theta-v_6)(C-\kappa_2)+\tfrac{(\theta-v_6)(h-\kappa_{10})(t-v_2)}{2},$$
$$\mathscr{A}_3 = \tfrac{K+\kappa_7}{t-v_2}+(\theta+v_7)(C+\kappa_3)+\tfrac{(\theta+v_7)(h+\kappa_{11})(t+v_3)}{2},$$
$$\mathscr{A}_4 = \tfrac{K+\kappa_8}{t-v_1}+(\theta+v_8)(C+\kappa_4)+\tfrac{(\theta+v_8)(h+\kappa_{12})(t+v_4)}{2}.$$

The graded mean integration value of the fuzzy number in Eq. (16) is obtained from Eq. (10) as

$$\begin{aligned}\varrho(\widetilde{\mathscr{A}}(\tilde{t})) = {} & \frac{1}{6}\left(\frac{K-\kappa_5}{t_4}+(\theta-v_5)(C-\kappa_1)+\frac{(\theta-v_5)(h-\kappa_9)t_1}{2}\right)\\ & +\frac{2}{6}\left(\frac{K-\kappa_6}{t_3}+(\theta-v_6)(C-\kappa_2)+\frac{(\theta-v_6)(h-\kappa_{10})t_2}{2}\right)\\ & +\frac{2}{6}\left(\frac{K+\kappa_7}{t_2}+(\theta+v_7)(C+\kappa_3)+\frac{(\theta+v_7)(h+\kappa_{11})t_3}{2}\right)\\ & +\frac{1}{6}\left(\frac{K+\kappa_8}{t_1}+(\theta+v_8)(C+\kappa_4)+\frac{(\theta+v_8)(h+\kappa_{12})t_4}{2}\right),\end{aligned} \tag{17}$$

where $t_i = t - \nu_i, i = 1,2; t_i = t + \nu_i, i = 3,4; C - \kappa_i > 0, i = 1,2; K - \kappa_i > 0, i = 5,6; h - \kappa_i > 0, i = 9,10$ and $\theta - \nu_i > 0, i = 5,6$.

The defuzzified value, $\varrho(\widetilde{\mathscr{A}}(\tilde{t}))$, is taken as the crisp estimate of fuzzy model in Eq. (12). In order to find the parameters which minimizes $\varrho(\widetilde{\mathscr{A}}(\tilde{t}))$, we have to solve the following partial derivatives of $\varrho(\widetilde{\mathscr{A}}(\tilde{t}))$ with respect to $\tilde{t} = (t_1, t_2, t_3, t_4)$ each equated to zero.

$$\begin{aligned}
&\frac{\partial\varrho(\widetilde{\mathscr{A}}(\tilde{t}))}{\partial t_1} = \frac{(\theta-\nu_5)(h-\kappa_9)}{12} - \frac{K+\kappa_8}{6t_1^2},\\
&\frac{\partial\varrho(\widetilde{\mathscr{A}}(\tilde{t}))}{\partial t_2} = \frac{(\theta-\nu_6)(h-\kappa_{10})}{6} - \frac{2(K+\kappa_7)}{6t_2^2},\\
&\frac{\partial\varrho(\widetilde{\mathscr{A}}(\tilde{t}))}{\partial t_3} = \frac{(\theta+\nu_7)(h+\kappa_{11})}{6} - \frac{2(K-\kappa_6)}{6t_3^2},\\
&\text{and}\quad \frac{\partial\varrho(\widetilde{\mathscr{A}}(\tilde{t}))}{\partial t_4} = \frac{(\theta+\nu_8)(h+\kappa_{12})}{12} - \frac{K-\kappa_5}{6t_4^2}.
\end{aligned} \tag{18}$$

Solving the above, we get

$$t_1 = \sqrt{\frac{2(K+\kappa_8)}{(\theta-\nu_5)(h-\kappa_9)}},\quad t_2 = \sqrt{\frac{2(K+\kappa_7)}{(\theta-\nu_6)(h-\kappa_{10})}},\quad t_3 = \sqrt{\frac{2(K-\kappa_6)}{(\theta+\nu_7)(h+\kappa_{11})}},\quad \text{and}\quad t_4 = \sqrt{\frac{2(K-\kappa_5)}{(\theta+\nu_8)(h+\kappa_{12})}}. \tag{19}$$

Note that $t_1 > t_2 > t_3 > t_4$ and hence the constraint $0 < t_1 < t_2 < t_3 < t_4$ is not satisfied. Hence, we adopt the Lagrangian method described in Section 3. For this, we convert the inequality constraint $t_2 - t_1 \geqslant 0$ into equality constraint $t_2 - t_1 = 0$ and minimize $\varrho(\widetilde{\mathscr{A}}(\tilde{t}))$ subject to $t_2 - t_1 = 0$. We have the Lagrangean function as

$$\mathscr{L}(t_1,t_2,t_3,t_4) = \varrho(\widetilde{\mathscr{A}}(\tilde{t})) - \lambda(t_2 - t_1), \tag{20}$$

where λ is the Lagrangian multiplier.

Taking the partial derivatives of $\mathscr{L}(t_1,t_2,t_3,t_4)$ with respect to t_1, t_2, t_3, t_4 and λ and equate to zero, we get

$$t_1 = t_2 = \sqrt{\frac{2(K+\kappa_8+2(K+\kappa_7))}{(\theta-\nu_5)(h-\kappa_9)+2(\theta-\nu_6)(h-\kappa_{10})}},\quad t_3 = \sqrt{\frac{2(K-\kappa_6)}{(\theta+\nu_7)(h+\kappa_{11})}},\quad \text{and}\quad t_4 = \sqrt{\frac{2(K-\kappa_5)}{(\theta+\nu_8)(h+\kappa_{12})}}. \tag{21}$$

Since $t_3 > t_4$, the above solution is not a local optimum. We get the similar result if repeat the procedure by selecting any one of the other inequality constraints. Hence, we convert two of the inequality constraints $t_2 - t_1 \geqslant 0$ and $t_3 - t_2 \geqslant 0$ as equality and minimize $\varrho(\widetilde{\mathscr{A}}(\tilde{t}))$ subject to $t_2 - t_1 = 0$ and $t_3 - t_2 = 0$. The Lagrangian function with multipliers λ_1 and λ_2 as

$$\mathscr{L}(t_1,t_2,t_3,t_4) = \varrho(\widetilde{\mathscr{A}}(\tilde{t})) - \lambda_1(t_2 - t_1) - \lambda_2(t_3 - t_2). \tag{22}$$

The solution obtained by setting the derivatives of $\mathscr{L}(t_1,t_2,t_3,t_4)$ in Eq. (22) with respect to $t_1, t_2, t_3, t_4, \lambda_1$ and λ_2 are all equal to zero is given by

$$\begin{aligned}
&t_1 = t_2 = t_3 = \sqrt{\frac{2(K+\kappa_8+2(K+\kappa_7)+2(K-\kappa_6))}{(\theta-\nu_5)(h-\kappa_9)+2(\theta-\nu_6)(h-\kappa_{10})+2(\theta+\nu_7)(h+\kappa_{11})}}\quad\text{and}\\
&\qquad t_4 = \sqrt{\frac{2(K-\kappa_5)}{(\theta+\nu_8)(h+\nu_{12})}}.
\end{aligned} \tag{23}$$

It may be noted from Eq. (23) that $t_1 = t_2 = t_3 > t_4$. That is, the solution given above is not local optimum as it does not satisfy the constraint $0 < t_1 \leqslant t_2 \leqslant t_3 \leqslant t_4$. We get the similar result if we repeat by selecting any two of the inequality constraints. Hence, the inequality constraints $t_2 - t_1 \geqslant 0, t_3 - t_2 \geqslant 0$ and $t_4 - t_3 \geqslant 0$ are converting into equalities, $t_2 - t_1 = 0, t_3 - t_2 = 0$ and $t_4 - t_3 = 0$.

The Lagrangian function with $\lambda_i i = 1,2,3,4$ multipliers is

$$\mathscr{L}(t_1,t_2,t_3,t_4) = \varrho(\widetilde{\mathscr{A}}(\tilde{t})) - \lambda_1(t_2 - t_1) - \lambda_2(t_3 - t_2) - \lambda_3(t_4 - t_3). \tag{24}$$

In order to minimize $\mathscr{L}(t_1,t_2,t_3,t_4)$ in Eq. (24), we take the partial derivatives of $\mathscr{L}(t_1,t_2,t_3,t_4)$ with respect to $t_1, t_2, t_3, t_4, \lambda_1, \lambda_2$ and λ_3 and equate to zero. Thus, we have $t_1 = t_2 = t_3 = t_4 = t^*$, where

$$t^* = \sqrt{\frac{2(K+\kappa_8+2(K+\kappa_7)+2(K-\kappa_6)+K-\kappa_5)}{(\theta-\nu_5)(h-\kappa_9)+2(\theta-\nu_6)(h-\kappa_{10})+2(\theta+\nu_7)(h+\kappa_{11})+(\theta+\nu_8)(h+\kappa_{12})}}. \tag{25}$$

Because the solution t^* satisfies all inequality constraints, the procedure terminates with t^* as the local optimum solution to the problem. Since the above local optimum solution is the only one feasible solution, it is the optimum solution of the model.

If the lower and upper modes are equal, the trapezoidal fuzzy number reduces to the triangular fuzzy number. In such case, the trapezoidal fuzzy numbers $\widetilde{C}, \widetilde{K}, \tilde{t}, \tilde{h}$ and $\tilde{\theta}$ are represented by the triangular fuzzy numbers, $\widetilde{C} = (C - \kappa_1, C, C + \kappa_4), \widetilde{K} = (K - \kappa_5, K, K + \kappa_8), \tilde{h} = (h - \kappa_9, h, h + \kappa_{12}), \tilde{t} = (t - \nu_1, t, t + \nu_4)$, and $\tilde{\theta} = (\theta - \nu_5, \theta, \theta + \nu_8)$, where $C > \kappa_1$, $K > \kappa_5, h > \kappa_9, t > \nu_1$ and $\theta > \nu_5$.

Fuzzy expected cost per unit time is represented as a triangular fuzzy number

$$\widetilde{\mathscr{A}}(\tilde{t}) = (\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3), \tag{26}$$

where

$$\mathscr{A}_1 = \frac{K-\kappa_5}{t+v_4} + (\theta - v_5)(C-\kappa_1) + \frac{(\theta-v_5)(h-\kappa_9)(t-v_1)}{2},$$
$$\mathscr{A}_2 = \frac{K}{t} + \theta C + \frac{\theta h t}{2},$$
$$\text{and } \mathscr{A}_3 = \frac{K+\kappa_8}{t-v_1} + (\theta + v_8)(C+\kappa_4) + \frac{(\theta+v_8)(h+\kappa_{12})(t+v_4)}{2}.$$

The defuzzified value of the fuzzy number in Eq. (26) is obtained from Eq. (11) as

$$\begin{aligned}\varrho(\widetilde{\mathscr{A}}(\tilde{t})) = &\frac{1}{6}\left(\frac{K-\kappa_5}{t_4} + (\theta - v_5)(C-\kappa_1) + \frac{(\theta-v_5)(h-\kappa_9)t_1}{2}\right) + \frac{4}{6}\left(\frac{K}{t} + \theta C + \frac{\theta h t}{2}\right)\\ &+ \frac{1}{6}\left(\frac{K+\kappa_8}{t_1} + (\theta + v_8)(C+\kappa_4) + \frac{(\theta+v_8)(h+\kappa_{12})t_4}{2}\right).\end{aligned} \tag{27}$$

Proceeding as in the case Eq. (17), the optimum time period is obtained as

$$t^* = \sqrt{\frac{2(6K+\kappa_8-\kappa_5)}{(\theta-v_5)(h-\kappa_9) + 4\,\theta\,h + (\theta+v_8)(h+\kappa_{12})}}. \tag{28}$$

6. Fuzzy EOT model with crisp time period

This is a replication of the model described in Section 5 with the only difference that the time period is regarded as crisp constant. Following the same notations as in Section 5, the fuzzy average cost per unit time with crisp time period is given by

$$\widetilde{\mathscr{A}}(t) = \frac{\widetilde{K}}{t} + \tilde{\theta}\widetilde{C} + \frac{\tilde{h}\,\tilde{\theta}\,t}{2}, \tag{29}$$

where

$$\frac{\widetilde{K}}{t} = \left(\frac{K-\kappa_5}{t}, \frac{K-\kappa_6}{t}, \frac{K+\kappa_7}{t}, \frac{K+\kappa_8}{t}\right), \tag{30}$$

$$\tilde{h}\,\tilde{\theta}\,t = ((\theta-v_5)(h-\kappa_9)t, (\theta-v_6)(h-\kappa_{10})t, (\theta+v_7)(h+\kappa_{11})t, (\theta+v_8)(h+\kappa_{12})t), \tag{31}$$

and, $\tilde{\theta}\widetilde{C}$ is the same as given by Eq. (14). The above Eqs. (30), (31) and (14) reduce the Eq. (29) into a trapezoidal fuzzy number as

$$\widetilde{\mathscr{A}}(t) = (\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4), \tag{32}$$

where the components

$$\begin{aligned}\mathscr{A}_1 &= \frac{K-\kappa_5}{t} + (\theta - v_5)(C-\kappa_1) + \frac{(\theta-v_5)(h-\kappa_9)t}{2},\\ \mathscr{A}_2 &= \frac{K-\kappa_6}{t} + (\theta - v_6)(C-\kappa_2) + \frac{(\theta-v_6)(h-\kappa_{10})t}{2},\\ \mathscr{A}_3 &= \frac{K+\kappa_7}{t} + (\theta + v_7)(C+\kappa_3) + \frac{(\theta+v_7)(h+\kappa_{11})t}{2} \quad \text{and}\\ \mathscr{A}_4 &= \frac{K+\kappa_8}{t} + (\theta + v_8)(C+\kappa_4) + \frac{(\theta+v_8)(h+\kappa_{12})t}{2}.\end{aligned} \tag{33}$$

As before, the defuzzified value of $\widetilde{\mathscr{A}}(t)$ is given by

$$\begin{aligned}\varrho(\widetilde{\mathscr{A}}(t)) = &\frac{1}{6}\left(\frac{K-\kappa_5}{t} + (\theta - v_5)(C-\kappa_1) + \frac{(\theta-v_5)(h-\kappa_9)t}{2}\right)\\ &+ \frac{2}{6}\left(\frac{K-\kappa_6}{t} + (\theta - v_6)(C-\kappa_2) + \frac{(\theta-v_6)(h-\kappa_{10})t}{2}\right)\\ &+ \frac{2}{6}\left(\frac{K+\kappa_7}{t} + (\theta + v_7)(C+\kappa_3) + \frac{(\theta+v_7)(h+\kappa_{11})t}{2}\right)\\ &+ \frac{1}{6}\left(\frac{K+\kappa_8}{t} + (\theta + v_8)(C+\kappa_4) + \frac{(\theta+v_8)(h+\kappa_{12})t}{2}\right).\end{aligned} \tag{34}$$

In order to find the value of t which minimizes $\varrho(\widetilde{\mathscr{A}}(t))$, we equate the derivative of Eq. (34) with respect to t to zero. That's

$$\frac{\partial\varrho(\widetilde{\mathscr{A}}(t))}{\partial t} = 0, \tag{35}$$

which reduces to

$$t = \sqrt{\frac{2(K+\kappa_8+2(K+\kappa_7)+2(K-\kappa_6)+K-\kappa_5)}{(\theta-v_5)(h-\kappa_9)+2(\theta-v_6)(h-\kappa_{10})+2(\theta+v_7)(h+\kappa_{11})+(\theta+v_8)(h+\kappa_{12})}}, \tag{36}$$

with the second-order derivative

$$\frac{\partial^2\varrho(\widetilde{\mathscr{A}}(t))}{\partial t^2} = \frac{(K-\kappa_5)+2(K-\kappa_6)+2(K+\kappa_7)+(K+\kappa_8)}{3t^3} > 0. \tag{37}$$

It should be noted that the optimum solution of the fuzzy model for crisp time period given by Eq. (36) is same as the solution of fuzzy model for fuzzy time period in Eq. (25).

In the case of the triangular fuzzy number, the fuzzy average cost per unit time is represented by

$$\widetilde{\mathscr{A}}(\tilde{t}) = (\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3), \tag{38}$$

where

$$\begin{aligned}
\mathscr{A}_1 &= \frac{K-\kappa_5}{t} + (\theta-v_5)(C-\kappa_1) + \frac{(\theta-v_5)(h-\kappa_9)t}{2},\\
\mathscr{A}_2 &= \frac{K}{t} + \theta C + \frac{\theta h t}{2},\\
\mathscr{A}_3 &= \frac{K+\kappa_8}{t} + (\theta+v_8)(C+\kappa_4) + \frac{(\theta+v_8)(h+\kappa_{12})t}{2}.
\end{aligned}$$

The defuzzified value of the fuzzy number in Eq. (38) is

$$\begin{aligned}
\varrho(\widetilde{\mathscr{A}}(\tilde{t})) &= \frac{1}{6}\left(\frac{K-\kappa_5}{t} + (\theta-v_5)(C-\kappa_1) + \frac{(\theta-v_5)(h-\kappa_9)t}{2}\right) + \frac{4}{6}\left(\frac{K}{t} + \theta C + \frac{\theta h t}{2}\right)\\
&\quad + \frac{1}{6}\left(\frac{K+\kappa_8}{t} + (\theta+v_8)(C+\kappa_4) + \frac{(\theta+v_8)(h+\kappa_{12})t}{2}\right),
\end{aligned} \tag{39}$$

and the optimum time period is given by

$$t = \sqrt{\frac{2(6K+\kappa_8-\kappa_5)}{(\theta-v_5)(h-\kappa_9)+4\,\theta\,h+(\theta+v_8)(h+\kappa_{12})}}. \tag{40}$$

The optimum time periods given by Eqs. (40) and (28) are the same.

Next we shall examine the model for the impact of fuzziness in the arrival rate alone.

7. EOT model with fuzzy arrival rate

We consider the model given by Eq. (3) with the arrival rate θ is represented by the trapezoidal fuzzy number $\tilde{\theta}$ in Section 5. The fuzzy expected cost function becomes

$$\begin{aligned}
\widetilde{\mathscr{A}}(t) &= \frac{K}{t} + \tilde{\theta}C + \frac{h\,\tilde{\theta}\,t}{2}\\
&= \frac{K}{t} + ((\theta-v_5)C, (\theta-v_6)C, (\theta+v_7)C, (\theta+v_8)C) + \left(\frac{(\theta-v_5)ht}{2}, \frac{(\theta-v_6)ht}{2}, \frac{(\theta+v_7)ht}{2}, \frac{(\theta+v_8)ht}{2}\right).
\end{aligned} \tag{41}$$

Eq. (41) reduces to the trapezoidal fuzzy number,

$$\widetilde{\mathscr{A}}(t) = (\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4), \tag{42}$$

where

$$\begin{aligned}
\mathscr{A}_1 &= \frac{K}{t} + (\theta-v_5)C + \frac{(\theta-v_5)ht}{2},\\
\mathscr{A}_2 &= \frac{K}{t} + (\theta-v_6)C + \frac{(\theta-v_6)ht}{2},\\
\mathscr{A}_3 &= \frac{K}{t} + (\theta+v_7)C + \frac{(\theta+v_7)ht}{2},\\
\text{and } \mathscr{A}_4 &= \frac{K}{t} + (\theta+v_8)C + \frac{(\theta+v_8)ht}{2}.
\end{aligned}$$

The defuzzified value of $\widetilde{\mathscr{A}}(t)$ using Eq. (10) is given by

$$\varrho(\widetilde{\mathscr{A}}(t)) = \frac{K}{t} + \frac{1}{6}\left((\theta - v_5)C + \frac{(\theta - v_5)ht}{2}\right) + \frac{2}{6}\left((\theta - v_6)C + \frac{(\theta - v_6)ht}{2}\right) + \frac{2}{6}\left((\theta + v_7)C + \frac{(\theta + v_7)ht}{2}\right) + \frac{1}{6}\left((\theta + v_8)C + \frac{(\theta + v_8)ht}{2}\right). \tag{43}$$

For necessary conditions of minima of $\varrho(\widetilde{\mathscr{A}}(t))$, we must have

$$\frac{\partial \varrho(\widetilde{\mathscr{A}}(t))}{\partial t} = 0, \tag{44}$$

which gives the value of t as

$$t = \sqrt{\frac{12K}{(\theta - v_5 + 2(\theta - v_6) + 2(\theta + v_7) + \theta + v_8)h}}. \tag{45}$$

The second-order derivative

$$\frac{\partial^2 \varrho(\widetilde{\mathscr{A}}(t))}{\partial t^2} = \frac{K}{3t^3} > 0. \tag{46}$$

When $\theta - v_6 = \theta + v_7$, the trapezoidal number $\tilde{\theta}$ becomes the triangular fuzzy number $\tilde{\theta} = (\theta - v_5, \theta, \theta + v_8)$. Then the fuzzy cost function in Eq. (41) reduces to the triangular fuzzy number

$$\widetilde{\mathscr{A}}(t) = (\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3), \tag{47}$$

where

$$\mathscr{A}_1 = \frac{K}{t} + (\theta - v_5)C + \frac{(\theta - v_5)ht}{2},$$
$$\mathscr{A}_2 = \frac{K}{t} + \theta C + \frac{\theta ht}{2},$$

and

$$\mathscr{A}_3 = \frac{K}{t} + (\theta + v_8)C + \frac{(\theta + v_8)ht}{2}.$$

The defuzzified value of $\widetilde{\mathscr{A}}(t)$ in Eq. (47) and the optimum time period are respectively given by

$$\varrho(\widetilde{\mathscr{A}}(t)) = \frac{K}{t} + \frac{1}{6}\left((\theta - v_5)C + \frac{(\theta - v_5)ht}{2}\right) + \frac{4}{6}\left(\theta C + \frac{\theta ht}{2}\right) + \frac{1}{6}\left((\theta + v_8)C + \frac{(\theta + v_8)ht}{2}\right) \quad \text{and} \tag{48}$$

$$t = \sqrt{\frac{12K}{(6\theta - v_5 + v_8)h}}. \tag{49}$$

The results of extensive numerical study performed are presented in the following section.

8. Numerical study

Consider an inventory system with crisp parameter values $C = \$300$ per unit, $K = \$50$ per setup/year, $h = \$20$ per unit/year and $\theta = 10$ units. The optimum time period and expected total cost obtained from Eqs. (3) and (4) are $t = 8.5$ months and $\mathscr{A}(t) = \$3141$.

We use the following sets of trapezoidal fuzzy numbers, based on arbitrary choices of κ_i and $v_j, i = 1, 2, \ldots, 12,\ j = 5, 6, 7, 8$; to represent the components of fuzzy models. The graded mean integration value (defuzzified value) and the corresponding percentage difference under fuzzy case (based on the defuzzified value) from the crisp value denoted by P_c for the component c, are also shown along with the fuzzy numbers.

C	$\varrho(C)$	P_C	h	$\varrho(h)$	P_h
(25,65,305,315)	180	−40	(2,3,21,22)	12	−40
(100,120,350,400)	240	−20	(2,7,25,30)	16	−20
(120,150,350,500)	270	−10	(5,10,24,35)	18	−10
(180,200,400,600)	330	+10	(10,15,25,42)	22	+10
(150,250,430,650)	360	+20	(12,15,31,40)	24	+20
(200,280,530,700)	420	+40	(12,18,35,50)	28	+40

K	$\varrho(K)$	P_K	θ	$\varrho(\theta)$	P_θ
(4, 8, 52, 56)	30	−40	(1, 1.5, 10.5, 11)	6	−40
(10, 20, 60, 70)	40	−20	(1, 4, 12, 15)	8	−20
(15, 35, 55, 75)	45	−10	(3, 6, 11, 17)	9	−10
(20, 40, 70, 90)	55	+10	(4, 7, 14, 20)	11	+10
(30, 40, 75, 100)	60	+20	(6, 8, 15, 20)	12	+20
(35, 45, 85, 125)	70	+40	(6, 9, 18, 24)	14	+40

The above fuzzy numbers reduce to the triangular fuzzy numbers when the lower and upper modes are equal to the crisp value.

The optimum time period t along with total cost for the fuzzy EOT model in Section 5 are computed from Eqs. (17), (25), (27) and (28). We use Eqs. (34), (36), (39) and (40) for the fuzzy EOT model with crisp time period. Under fuzzy arrival rate, Eqs. (43), (45), (48) and (49) give the optimum t and the total cost.

Tables 1–4 reveal that the optimum time for fuzzy model with fuzzy time period is as same as fuzzy model with crisp time period.

Table 3 shows variations in the optimum decision variable t and the expected total cost due to fuzziness in all the components (except t) of the model. It reveals that the optimum value t and the average total cost are highly sensitive to the level of fuzziness in the components. The percentage changes in the optimum costs are found to increase with the increasing percentage changes in the level of fuzziness of the components. The positive change in the components due to fuzziness decrease t while t increases for the negative changes.

Table 5 exhibits the variations in the optimum values of the decision variable t and expected total cost with respect to changes in the levels of the fuzzy mean arrival rate. The optimum decision variable t considerably decreases while the optimum average total cost increases with respect to increase in the fuzzy mean arrival rate (in terms of defuzzified value). Note

Table 1
Optimum policy under fuzzy EOT model for trapezoidal fuzzy numbers.

P_C	P_h	P_K	P_θ	t^* (Months)	$\widetilde{\mathscr{A}}(\tilde{t})$
−40	−40	−40	−40	8.7	(31, 110, 3354, 3630)
−20	−20	−20	−20	7.9	(116, 520, 4390, 6255)
−10	−10	−10	−10	7.8	(388, 973, 4020, 7009)
00	00	00	00	8.5	(3141, 3141, 3141, 3141)
+10	+10	+10	+10	7.3	(765, 1498, 5822, 12,403)
+20	+20	+20	+20	7.1	(972, 2103, 6714, 13,406)
+40	+40	+40	+40	6.5	(1284, 2647, 9868, 17,356)

Table 2
Optimum policy under fuzzy EOT model for triangular fuzzy numbers.

κ_1, κ_4	κ_5, κ_8	κ_9, κ_{12}	v_5, v_8	t^* (Months)	$\widetilde{\mathscr{A}}(t)$
275, 15	18, 2	46, 6	9, 1	9.12	(31, 3141, 3630)
200, 100	18, 10	40, 20	9, 5	8.78	(116, 3141, 6255)
180, 200	15, 15	35, 25	7, 7	8.34	(388, 3141, 7009)
0, 0	0, 0	0, 0	0, 0	8.50	(3141, 3141, 3141)
120, 300	10, 22	30, 40	6, 10	7.96	(765, 3141, 12,403)
150, 350	8, 20	20, 50	4, 10	8.09	(972, 3141, 13,406)
100, 400	18, 20	15, 75	4, 14	7.91	(1284, 3141, 17,356)

Table 3
Optimum policy under fuzzy EOT model with crisp time period for trapezoidal fuzzy numbers.

P_C	P_h	P_K	P_θ	t (Months)	% Change in t	$\varrho(\widetilde{\mathscr{A}}(t))$	% Change in $\varrho(\widetilde{\mathscr{A}}(t))$
−40	−40	−40	−40	8.7	2.35	1765	43.8
−20	−20	−20	−20	7.9	7.06	2698	14.1
−10	−10	−10	−10	7.8	8.24	2822	10.2
00	00	00	00	8.5	0.00	3141	0.00
+10	+10	+10	+10	7.3	14.1	3464	10.3
+20	+20	+20	+20	7.1	16.5	5435	73.0
+40	+40	+40	+40	6.5	23.5	7278	131.7

Table 4
Optimum policy under fuzzy EOT model with crisp time period for triangular fuzzy numbers.

κ_1, κ_4	κ_5, κ_8	κ_9, κ_{12}	v_5, v_8	t (Months)	% Change in t	$\varrho(\widetilde{\mathscr{A}}(t))$	% Change in $\varrho(\widetilde{\mathscr{A}}(t))$
275,15	18,2	46,6	9,1	9.12	7.3	2339	25.5
200,100	18,10	40,20	9,5	8.78	3.3	2772	11.8
180,200	15,15	35,25	7,7	8.34	1.9	3102	1.24
0,0	0,0	0,0	0,0	8.50	0.0	3141	0.00
120,300	10,22	30,40	6,10	7.96	6.4	3676	17.0
150,350	8,20	20,50	4,10	8.09	4.8	3830	21.9
100,400	18,20	15,75	4,14	7.91	6.9	4265	35.8

Table 5
Optimum policy under fuzzy arrival rate for trapezoidal fuzzy numbers.

P_θ	t (Months)	% Change in t	$\varrho(\widetilde{\mathscr{A}}(t))$	% Change in $\varrho(\widetilde{\mathscr{A}}(t))$
−40	10.95	28.8	1910	39.2
−20	9.487	11.7	2527	19.6
−10	8.945	5.29	2835	9.80
00	8.500	0.00	3141	0.00
+10	8.090	4.82	3447	9.80
+20	7.746	8.87	3755	19.6
+40	7.171	15.6	4367	39.2

Table 6
Optimum policy under fuzzy arrival rate for triangular fuzzy numbers.

v_5	v_8	t (Months)	% Change in t	$\varrho(\widetilde{\mathscr{A}}(t))$	% Change in $\varrho(\widetilde{\mathscr{A}}(t))$
9	1	9.12	7.3	2732	13.0
9	5	8.78	3.3	2947	6.18
7	7	8.50	0.0	3141	0.00
0	0	8.50	0.0	3141	0.00
6	10	8.22	3.3	3346	6.53
4	10	8.09	4.8	3448	9.77
4	14	7.86	7.5	3653	16.3

that the percentage changes in t are nearly half and those in the average total cost are nearly equal to the percentage changes in the mean arrival rate at various levels.

When the trapezoidal fuzzy number collapses to the triangular fuzzy number (Tables 4 and 6), the sensitiveness in the optimum decision variable and the average total cost is comparatively less due to fuzziness in the components. The case, where the lower limit v_5 increases while the upper limit v_8 decreases, the optimum expected total cost increases when fuzziness is allowed in the arrival rate (Table 6). When $v_5 = v_8$ ($\tilde{v}$ is symmetric), fuzziness in the arrival rate has no impact as it reduces to the crisp case.

9. Conclusion

We conclude that the solution of the EOT model with all components fuzzy is the same as those under the fuzzy model with crisp time period. Hence, the fuzziness in the optimum period has no much relevance. The decision variable and the average total cost are highly sensitive due to fuzziness in the cost components and arrival rate when considered together. Under fuzzy arrival rate, the percentage change due to fuzziness cause approximately equal percentage changes in the total cost and the decision variable is nearly half of the changes in the component. The case where the upper and lower modes of the trapezoidal number coincides, the sensitivity in the optimum decision variable and average total cost reduces. The decision maker should adopt a better trade of judgement for accounting flexibility in the characteristics of the model in order to tackle the uncertainty which fits to the real situations.

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